Diploma Thesis
LODZ UNIVERSITY OF TECHNOLOGY
FACULTY OF MECHANICAL ENGINEERING

Programme of Study: Mechanical Engineering and Applied Computer Science

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Performance analysis and optimization of wind turbine behaviour for various wind conditions using the Blade Element Momentum Method

Analiza wydajności i optymalizacja zachowania turbiny wiatrowej dla różnych warunków wietrznych przy użyciu metody Blade Element Momentum

Lodz 2021

Acknowledgments

I would like to express my gratitude for the tremendous support I received while working on this engineering thesis.

At the outset, I would like to thank my superiors:

Dr Vladimir Leble for his extensive expertise and knowledge, which helped a lot in this work and my understanding of aerodynamics and wind energy industry

and

Dr Grzegorz Liśkiewicz for awakening my passion for programming, which after his course became my hobby, and for all the useful insights that turned out to be very helpful during the entire process of completion of this work.

My special thanks likewise go to the Lodz University of Technology and the entire academic staff for giving me this opportunity to develop my skills and gain an important education that allows me to achieve things that were beyond my horizon of possibilities before.

I also have a lot of gratitude for all the companies that have made their valuable data public so that students like me can benefit from it. I would especially like to mention CREADIS, which agreed to provide operating parameters for various environmental conditions for the DTU 10MW wind turbine, which is the cornerstone of my thesis.

I would also like to thank my friends and loved ones for the kind words of encouragement that have never failed to make my day brighter and have kept me motivated throughout my studies.

Abstract

The main goal of this thesis is to provide useful data together with its analysis that will allow us to adjust the wind turbine behaviour to the various environmental conditions. This will be done in such a way that it will maximize the generated amount of power, while avoiding the risk of breakdown of the turbine at the same time. The turbine behaviour will be adjusted by designing the optimized database for the controller, that can be used as an input for the scripts managing instructions for the pitch angle and rotor rotation.
While the presented solution will work for various models the test case was chosen for the DTU 10MW offshore wind turbine.
In order to achieve this goal firstly the two main theories will be introduced and explained- the Momentum Theory and the Blade Element Theory. This will make it possible to derive a combination of these two in the form of the Blade Element Momentum Theory (BEM) which will be then used to create an algorithm calculating power and thrust for a given wind turbine working under given conditions. This algorithm will be then used multiple times in an iterative process running through different simulated conditions and finding the best wind turbine behaviour for each situation. The collected data will be then presented and discussed.
Since this problem offers a lot of freedom while choosing methods of solving, different approaches will be utilized and discussed. The pros and cons of usage of additional correctional algorithms will also be discussed and in order to ensure that the provided solution is reliable the data will be compared to the data collected by the Creadis company. Additionally, the results will be discussed with an expert in the field of renewable energy.
Finally, the challenges encountered during the solution finding process will be described together with the conclusions on how to overcome them.

Głównym celem niniejszej pracy jest dostarczenie użytecznych danych wraz z ich stosowną analizą, która pozwoli na dostosowanie zachowania turbiny wiatrowej do różnych warunków środowiskowych w taki sposób, aby generowała ona jak największą ilość energii przy jednoczesnym uniknięciu ryzyka awarii w tym samym czasie. Przedstawione rozwiązanie będzie uniwersalne i sprawdzi się w różnych modelach turbin wiatrowych, ale przypadek testowy został wybrany dla turbiny wiatrowej DTU 10MW.
Aby osiągnąć założony cel, najpierw zostaną przedstawione i wyjaśnione dwie główne teorie: Momentum Theory i Blade Element Theory. Umożliwia to wyprowadzenie teorii Blade Element Momentum (BEM), która następnie posłuży do stworzenia algorytmu obliczającego moc i ciąg dla danej turbiny wiatrowej pracującej w określonych warunkach. Algorytm ten zostanie następnie wykorzystany wielokrotnie w procesie iteracyjnym, w którym symulowane są różne warunki wietrzne i znajdzie on najlepsze zachowanie turbiny wiatrowej dla każdej symulowanej sytuacji. Zebrane dane zostaną następnie zaprezentowane i omówione.
Omówione zostaną również plusy i minusy stosowania dodatkowych algorytmów korekcyjnych oraz w celu zapewnienia rzetelności dostarczonego rozwiązania dane zostaną porównane z danymi zebranymi przez firmę Creadis. Dodatkowo wyniki zostaną omówione z ekspertem w dziedzinie energii odnawialnej.
Na koniec zostaną opisane wyzwania napotkane podczas procesu poszukiwania rozwiązania wraz z wnioskami, jak je przezwyciężyć.

Nomenclature

a [-] - axial induction factor

a’ [-] - angular induction factor

B [-] - blades number

c [m] - aerofoil chord length

c_n [-] - normal aerodynamic coefficient

c_t [-] - tangential aerodynamic coefficient

Cl [-] - lift coefficient

Cd [-] - drag coefficient

Cp [-] - power coefficient

D [N] - drag force

L [N] - lift force

F [-] - tip-loss factor

F_n [N] - normal force

F_t [N] - tangential force

$\dot{m}$ [$\frac{kg}{s}$] - mass flow rate

N [-] - number of discretized blade elements

P [W] - power

Q [-] - tip loss correction factor

r [m] - radius/radial position

R [m] - radius of a wind turbine rotor

P_{total} [W] - total power output

p [$\frac{N}{m^2}$] - pressure

T [Nm] - torque

T_h [N] - thrust

V [$\frac{m}{s}$] - absolute velocity of the wind flow

W [$\frac{m}{s}$] - relative velocity of the wind flow

V_1 [$\frac{m}{s}$] - velocity far upstream of the turbine in streamtube in the Momentum theory

V_2 [$\frac{m}{s}$] - velocity right before entering the turbine in streamtube in the Momentum theory

V_3 [$\frac{m}{s}$] - velocity right after leaving the turbine in streamtube in the Momentum theory

V_4 [$\frac{m}{s}$] - velocity far downstream of the turbine in streamtube in the Momentum theory

U_t [$\frac{m}{s}$] - flow transversal component in the rotor plane

U_n [$\frac{m}{s}$] - flow longitudinal component normal to the rotor

α [rad] - angle of attack/incidence

β [rad] - relative flow angle onto blade

θ [rad] - aerofoil twist angle

λ [-] - tip speed ratio

λ_r [-] - local tip speed ratio

ρ [$\frac{kg}{m^3}$] - fluid density

η [-] - efficiency

σ [-] - local solidity factor

Ω [$\frac{rad}{s}$] - rotor rotational velocity

ω [$\frac{rad}{s}$] - wake rotational velocity

1. Introduction

When discussing wind turbines and their economical and ecological impact it is important to take into consideration the complexity of this very broad topic. This chapter will describe concepts related to wind turbines and clean energy, and the impact this technology has on both the market and the environment. Additionally, an overview of the principles of generating wind energy will be presented, along with an explanation of the physical laws affecting this process.

1.1 The need for better solutions- why do we have to keep increasing the efficiency?

The recently increasing need for clear renewable energy forces us to come up with better and better ways for its extraction. This need comes mostly from the rapidly advancing climate changes and inevitable fossil fuel depletion that is expected to happen approximately in the next 40 years [1].

In a way, wind energy is actually one of the forms of solar energy although for statistical reasons it is not classified as such. When the sun is heating different layers of the atmosphere it creates temperature irregularities that together with the Earth's rotation and the Earth's surface non-uniformity causes air to translocate, which then results in winds blowing [2].

Every 24 hours, wind generates enough kinetic energy to produce roughly 35 times more electricity than we are using each day.

Since the wind supply cannot be realistically exhausted there is no fear that this energy source exhaustion will ever happen to the wind power industry. It makes investing into this field safer because we know it will always be an available solution.

In addition to that there is no pollution coming out of wind farms which gives them an edge in comparison with the classical, coal based power plants. While power plants are emitting huge amounts of dangerous to humans nitrogen oxides and sulfur dioxides, the wind turbines do not emit anything to the atmosphere. There are no acid rains caused by the wind farms and they cause no smog and they do not emit any greenhouse gases.

This advantage helps to avoid any potential costs associated with the prevention of pollution and enormous fines often imposed on other energy generating industries.

While it is clear that there is still a huge potential in the wind energy extraction industry there still are some challenges to overcome before this clear form of energy becomes available on a larger scale. For instance it still has to compete with cheaper and faster non-renewable energy production methods and it often is a huge investment to start a wind farm that will pay for itself in a relatively short time. Another problem is that wind turbine blades can not be properly recycled, so exploited parts are being piled up in landfills instead. This of course increases the overall cost, creates another logistical problem to manage and makes a negative impact on the environment which goes against the concept of clean energy.

That's why it is important to keep looking for better and better solutions that will increase the efficiency and therefore profitability of those farms. Even a small change in the efficiency will result in huge amounts of extra power generated annually and every little bit of extra clean power is needed if we want to preserve earth in a relatively good condition for the years to come.

1.2 Principle of operation of wind turbines- how do they work?

Modern wind turbines were preceded by windmills that were using wind energy to crush and grind grain into flour that were used since the 7th century. The first wind turbine that was generating electricity was invented in 1887. Since then, the design of wind turbines changed continuously from bulky, wide structures to tall and slim towers we are used to seeing.

Principle of operation

The most basic definition of the principle of work of wind turbines is that the wind turbines convert kinetic energy of moving air particles to electrical energy. The torque is being generated when the air flowing over the blades creates a lift force. The wind turbine blade is oriented in such a way that there is a component of the lift that induces rotation of the blades. This means that kinetic energy of the wind is being converted into rotational energy, which in turn is being converted into electrical energy in the generator. Of course not all wind turbines are generating the same amount of electricity- there are 3 main factors that determine the power output of a wind turbine:

- blade orientation,
- blade design,
- speed of wind

Below each of those elements is summarized in brief.

Blade orientation

The first is the blade orientation, which is determined by the programmed behaviour managed by the controller. The process of adjusting the wind turbine orientation is called yawing, therefore this orientation is often referred as the yaw. Another thing is the pitch angle which describes the angle of the blades.Although older and less sophisticated wind turbines can have their blades bolted directly to the hub, the more modern and expensive ones have blades bolted to the pitch bearing, which allows them to adjust their angle of attack. The advantage of this solution is that it allows them to avoid the aerodynamic stall by adjusting the pitch angle of the blade to the variations in wind speed. This pitch adjustment is done either by the electrical or mechanical systems connected to the controller.
While the yaw ensures that the wind turbine is always facing in the advantageous direction the pitch angle adjustment is the best method of increasing or limiting the angle of attack and in doing so changing drastically the power output of the wind turbine.

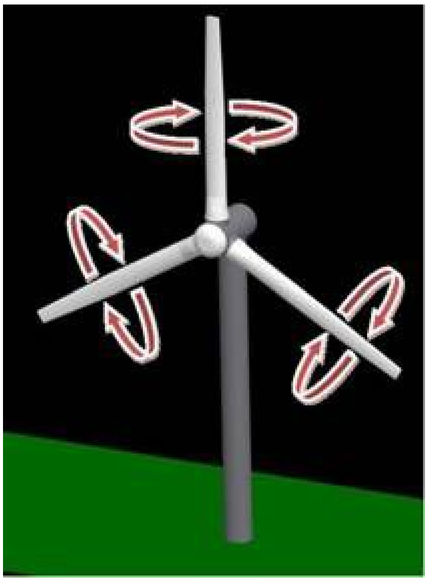

Figure 1 Pitch adjustment [3]

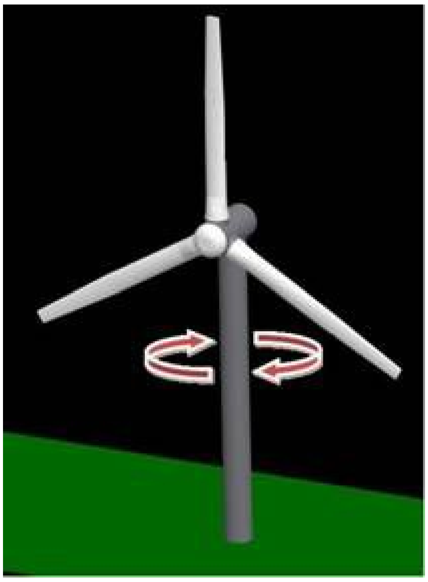

Figure 2 Yaw adjustment [3]

Wind turbines are equipped with very precise wind sensors and computer systems (aforementioned controller) that automatically adjust the pitch and the yaw to optimize the amount of energy harvested.

Blade design - material

Another very important factor that has a great influence over the wind turbine performance is the aerodynamic design of the blades. Wind turbine rotor blades are also exposed to wind and gravitational loading, therefore the ultimate loading failure mode has to be taken into serious consideration and prevented or at least counteracted by the blades design.
Because of these mechanical strength related challenges and also since the blades are meant to work in very severe weather conditions, often surviving rains, storms and lightning strikes, they are usually made out of a strong combination of fiberglass and resin laytes. This material choice fits its purpose well, because it needs to have relatively high fatigue resistance to withstand cyclic loading, high fracture toughness, and low weight and density to reduce gravitational forces acting on the rotor and the nacelle. High stiffness of the fiberglass composite blades is also a very important factor because it helps maintain the optimal shape and orientation of the blade with clearance with the tower. Another important factor is the price and the availability of the material. It is especially important when we consider the amount and the size of the produced blades, and the recently rapidly increasing demand for them. It is estimated that in 2001 it took about 50 million kilograms of fiberglass laminate to maintain the annual production of the wind turbine blades.
Considering those reasons, the fiber-reinforced composite is the best material for the mass production of the wind turbine blades. While it is possible to use other materials, it is simply not cost effective, except in some fairly specific and unusual situations. The blades made out of metal are too heavy and are rather vulnerable to fatigue, ceramic blades do not have high fracture toughness and traditional polymers are not stiff enough for this kind of application.

Blade design - shape and arrangement

Similarly to the wings of an airplane, the blades of the wind turbine must have good aerodynamic properties. The shape of the blade outer cross-section is called an airfoil and in most modern cases it is cambered similarly to the cross-section of the airplane wings. The reason that stands behind this design is the fact that the wind travels faster over the curved surfaces and in doing so it creates a region with lower pressure above the blade. This difference in pressure creates an aerodynamic force that acts on the blade pulling it upwards. Blade designs also include a twist angle which affects the angle of attack. The outermost parts of the blade are harvesting significantly more energy than the parts that are closest to the hub and therefore those inner blade sections are having lower torque and they do not really contribute much to the power output of the wind turbine. For this reason the aerodynamic design of the inner blade section is often disregarded. The airfoil of the first few sections of the wind turbine that was chosen as the test case for this engineering thesis is also cylindrical. The loss in power output in those cylindrical airfoil sections is negligible in comparison to the huge gains in easier manufacturability, reduced weight and saved material.
Usually, most wind turbines are equipped with 3 blades (as does the test case that was chosen for the study of this work). More blades means more materials needed to manufacture them, more transportation issues and more installation costs. On the other hand, installing only 1 blade would make the whole construction very unbalanced, and installing 2 blades would make it unstable when turning to face changing wind direction [4]. Another important difference is that 3 bladed rotor has the same moment of inertia with respect to axes perpendicular to axis of rotation, which is not the case for 2 bladed rotor.

Speed of the wind

The third factor that influences the power output is the speed of the wind. After all, if the air is not moving then there is no kinetic energy to be extracted from it. Many regions are naturally more windy than the others (mostly due to the geographical reasons) and those locations are often perfect sites for wind farms. Since the wind speed typically increases with the altitude, wind turbines are often mounted on towers that are over 100m tall.
As it was stated, the electricity is being generated in the generator.
Unless the wind turbine is equipped with the direct drive, the rotation of the blades cannot be directly coupled to the generator, because the rate at which they are rotating is often too low. Therefore before the rotational energy is being transported to the generator it first goes via the shaft to the gearbox where the rotational ratio is being amplified in the planetary gear system. Due to the very high number of moving elements, the gearbox is one of the most maintenance demanding parts of a wind turbine. The alternative to this problem is the aforementioned direct drive which allows the generation of electricity at lower speeds. Direct drive wind turbines however are usually using permanent magnets which are very expensive due to their rarity, so most wind turbines are using gearboxes instead [5].
The interior of the wind turbine is also equipped with a brake which allows to stop the rotation of the blades during the storm when the wind conditions are too excessive for the wind turbine to operate safely. Those inner parts are stored in the nacelle (mounted on top of the tower) which is a solid, hollow shell. The tower and the nacelle are typically made out of steel partially due to transportation reasons, and they are interconnected with flanges and screws when they are assembled.

Look into the future

Since the wind energy market is very rapidly developing, there is a lot of scientific effort to improve wind turbines and make them as efficient as possible. While it is hard to confidently predict what will be the result of this research it is known that new light, yet durable materials are being investigated for various parts of the wind turbines that will decrease the costs associated with the manufacturing, construction and the maintenance of the turbines. There are also new exciting designs of the wind turbines that are currently being developed and introduced to the market such as bladeless vortex wind turbines that instead of capturing the energy via the propeller motion, capture the energy from the aeroelastic oscillation movement of the device [6]. Another interesting design that could potentially see commercial usage in the days to come is the Wind Lens- a shrouded six-bladed wind turbine that is now being tested off the coast of Japan [7].
Although there is no shared belief how the wind farms of the future will look like, the main goal of the ongoing research is to maximise the efficiency which is sooner or later bound to reach a certain threshold for the reasons described in the next section.

1.3 The Betz limit- how efficient can we ever hope to make wind turbines?

As it is always the case in physics, according to the second law of thermodynamics there exists no method of energy conversion that is 100% efficient. The wind carries a finite amount of energy and there exists a very specific amount of it that can be extracted by a wind turbine and transformed into electricity. The air is made out of a great number of moving particles each of which has its mass and velocity. Thus the kinetic energy of each singular particle can be described as:

$$E_{kinetic_i} = \frac{1}{2} m_i v_i^2 \qquad (1.3.1)$$

Since the air is made out of a system of such particles this equation also holds true when treated as such. Therefore the kinetic energy of air has a form:

$$E_{kinetic} = \sum_{i=1}^{n} E_{kinetic_i} \qquad (1.3.2)$$

$$E_{kinetic} = \frac{1}{2} m\, v^2 \qquad (1.3.3)$$

Therefore one can conclude that the faster moving air has much more energy than slow moving air. The energy also increases with the mass of the air. The mass of the air intersecting the turbine area can be described as:

$$m = \rho \cdot V \qquad (1.3.4)$$

The density ρ varies with the temperature and the altitude. The interesting conclusion that can be drawn from this information is that wind turbines that are located in colder areas and at lower altitudes will generate more power than they would in warm areas at high altitudes, because each volume of air would carry a bigger amount of energy, assuming of course that the wind speeds would be the same in those 2 compared locations.

The blades are sweeping a disk in their path when they are rotating. This will be further described in the section 2.1 but for the purpose of the analysis of the Betz law let's call the area of this disk A . We can notice then, that the cylinder of air with thickness t will have mass:

$$Area = A$$

$$m = A \cdot \rho \cdot t \qquad (1.3.5)$$

Now let's notice that if air is travelling through that disk with the wind speed v then the mass flow rate is:

$$\dot{m} = v \cdot t \cdot A \cdot \rho \quad [\frac{kg}{s}] \qquad (1.3.6)$$

kinetic energy of the air with respect to time that passes through this disk is the power:

$$P = \frac{E}{t} \qquad (1.3.7)$$

Equation (1.3.7) combined with equation (1.3.3) gives:

$$P = \frac{\frac{1}{2}mv^2}{t} \qquad (1.3.8)$$

and since:

$$m = \frac{\dot{m}}{t} \qquad (1.3.9)$$

We can couple equation (1.3.8) with equation (1.3.9) and obtain the formula for the total power P of air with the wind speed v flowing through disk area A:

$$P = \frac{1}{2} \cdot \rho \cdot A \cdot v^3 \cdot t \qquad (1.3.10)$$

The wind that blows through the disk swept out by the turbine rotor comes from way upstream of the turbine and extends way downstream.

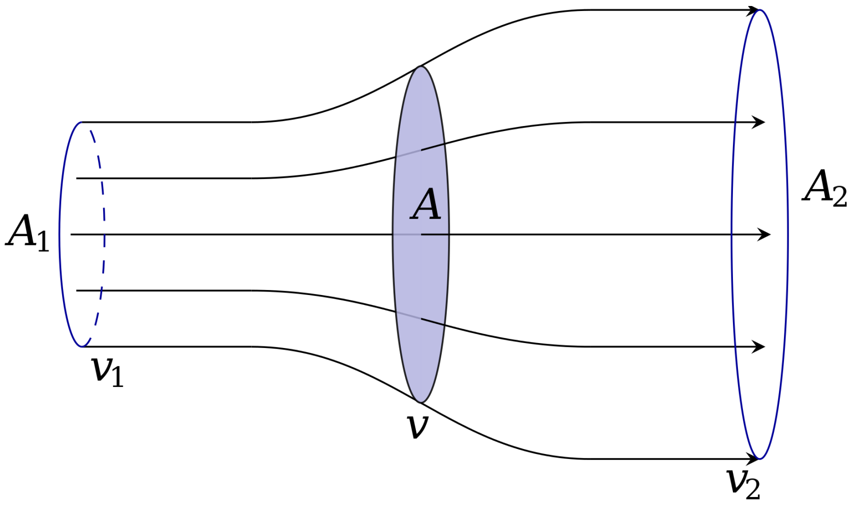

Figure 3 wind stream in annular stream tube [8]

Some of the energy of the air is taken out of the air by the turbine- that's the whole point of the wind turbine. As it was proven the amount of energy in the air depends on its mass and its velocity squared and since the turbine does not have the capability to transfer entire available wind kinetic energy - every particle of the air that gets inside the rotor disk also is able to leave it. The conclusion that can be drawn from this is that in the case of an uniform inflow in any location in the stream tube the mass is the same. So, since the kinetic energy of the air is reduced but the mass remains constant then we know that it's speed must slow down after energy is being extracted. Since the mass does not change all along the stream tube, but the speed changes, then the width of the stream tube has to change. We can observe this in equation 1.3.6 when we notice that when v decreases A has to increase. That's how we know it is wider downwind of the turbine when the wind speed is low, while it is narrower upwind where the wind speed is relatively higher.

So what does all that have to do with the limit of energy that can be harvested by the wind turbine? If all the energy would be taken away by the wind turbine then the downstream velocity would be $v_2 = 0$. Without any velocity the air particles would stop flowing any further and they would just all pile up right behind the turbine which is not possible, because if the wind would cease to move at the end of the turbine then there would be no possibility for the fresh inflow of wind to get in. Therefore in order to keep the wind flowing through, there has to be some wind movement, however small, downstream with positive velocity.

So we know that there is some kind of limit describing how much energy can be extracted by the wind turbine and that's how we arrive at the concept of the Betz Law (which can be applied to incompressible fluids). In 1919 Albert Betz published work that described the limit which indicates the maximum power that can be idealistically extracted from the wind, regardless of what kind of design the wind turbine has [9].

Betz proved that the maximum amount of energy that we can ever hope to extract from the stream tube is 59.3% . It is the maximum efficiency that any wind turbine can have.

The efficiency can be obviously increased by improving the design and the behaviour of the wind turbines (which is one of the goals of this work) but it will never ever, in any case exceed the Betz limit. There exists a misconception that the addition of shrouds or diffusers can elevate the efficiency above Betz limit, however after more detailed analysis it was concluded that the Betz Law is not violated in those cases for reasons beyond the topic of this engineering thesis [10].

One has to remember that the Betz limit can never be fully reached in reality, because there are always some additional energy losses associated with moving elements (for example blades moving through the air are meeting resistance, and transfer of energy between shaft and gearbox will never be perfectly efficient etc.). Modern technologically advanced wind turbines have efficiency close to 50% so we are really not that far away from the (unreachable) perfection.
To derive this law one can start from the assumption that average value of the wind speed through the rotor area is the average of the undistributed wind speed before the wind turbine (v_1), and the wind speed after it leaves the rotor plane (v_2). This assumption was proven to be true also by Betz. Therefore we arrive at:

$$v_{avg} = \frac{v_1+v_2}{2} \qquad (1.3.11)$$

The mass m_{air} of the air with density ρ_{air} that travels through the rotor can be expressed as:

$$m_{air} = \rho_{air} V = \rho_{air} A\, H \qquad (1.3.12)$$

where A is the area swept by the rotor and H is the height of the cylinder that describes the volume of operation of the rotor. Since:

$$v_{avg} = \frac{s}{t} = \frac{H}{t} \qquad (1.3.13)$$

$$H = v_{avg}\, t \qquad (1.3.14)$$

We arrive at the formula for the mass of the airstreaming through the rotor in time:

$$\frac{m_{air}}{t} = \rho_{air} A\, v_{avg} \qquad (1.3.15)$$

Therefore the m_{air} that gets through the rotor in 1 second can be expressed as:

$$m_{air} = \rho_{air}\, A \frac{v_1+v_2}{2} \qquad (1.3.16)$$

According to Newton's second law, the power extracted from the moving air by the rotor equals the mass times the drop in the wind speed squared:

$$P = \frac{1}{2} m_{air}(v_1^{\,2} - v_2^{\,2}) \qquad (1.3.17)$$

When combining equation (1.3.16) with (1.3.17) the following expression is derived:

$$P = \frac{\rho_{air}}{4}(v_1^{\ 2} - v_2^{\ 2})\,(v_1 + v_2)\,A \qquad (1.3.18)$$

The next step is to compare this power expression (1.3.18) with the total power in the undistributed wind streaming through the same swept area A, this time with no rotor taking energy from the wind. This total power will be denoted as P_0:

$$P_0 = \frac{\rho_{air}}{2}\,A\,v_1^{\ 3} \qquad (1.3.19)$$

The ratio of these powers is:

$$\frac{P}{P_0} = \frac{1}{2}\frac{(v_1^{\ 2}-v_2^{\ 2})\,(v_1+v_2)}{v_1^{\ 3}} \qquad (1.3.20)$$

$$\frac{P}{P_0} = \frac{1}{2}\left(1 - \left(\frac{v_2}{v_1}\right)^2\right)\left(1 + \frac{v_2}{v_1}\right) \qquad (1.3.21)$$

The ratio $\frac{P}{P_0}$ can be plotted as a function of $\frac{v_2}{v_1}$:

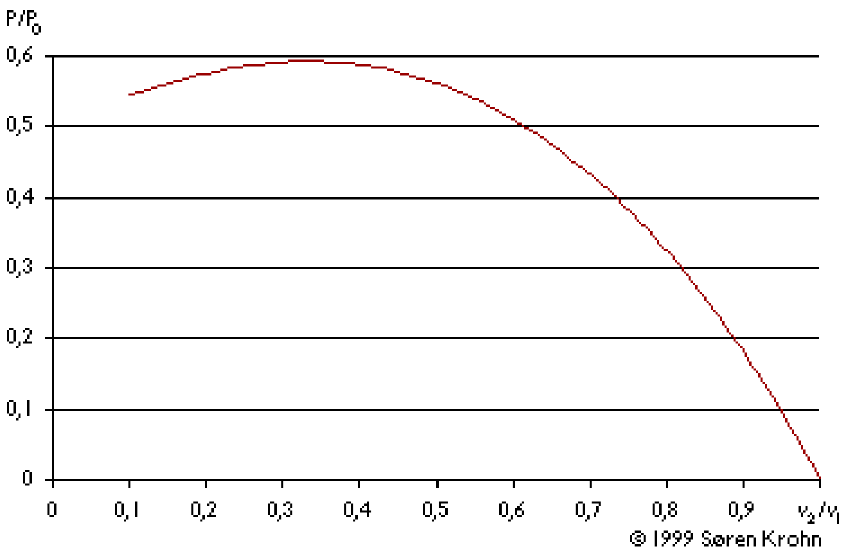

Figure 4 the ratio of $\frac{P}{P_0}$ [11]

From the analysis of the graph one can conclude that the function reaches its maximum value for the argument of $\frac{v_2}{v_1} = \frac{1}{3}$.

The maximum value of $\frac{P}{P_0}$ at this point is 0.593 which is the Betz Limit. It means that the maximum value that can be extracted from the air by the wind turbine rotor is 59.3% of the total power of the streaming wind [11].

1.4 Onshore vs offshore wind farms- which is better?

Regarding the location at which the wind turbines are being installed, we distinguish two main types of wind turbines: onshore and offshore. Onshore wind turbines are located on the land and the offshore wind turbines are being placed over open sea water. Both onshore and offshore wind energy extraction farms are among the most promising technologies that can provide clean energy and achieve the very important goal of greatly reducing the carbon footprint that burdens other sectors of the energy industry. While there is a difference in size, because offshore wind turbines are more massive, the mechanical principle of operation remains the same in both cases.

From the economical point of view it is significantly cheaper to install wind farms on land than it is to do it on water. To place an offshore wind turbine it is necessary to install special platforms connected with underwater cables and it is also expensive to transport all the parts to the location and to bring the human maintenance team using a ship or preferably a helicopter. All those logistical dilemmas often scare away smaller investors who opt for onshore wind farms instead. However it is important to mention that the offshore wind farms have the advantage of being subjected to higher wind speeds that are more predictable than on land. Therefore offshore wind turbines power generation is more efficient and more reliable on a wider scale.

In 2019, a new record was set for the largest wind turbine ever built. The enormous rotor diameter of 220 meters provides enough power each year to power 16 000 households [12]. The GE Haliade-X 12MW wind turbine that operates off the coast of Netherlands is the perfect example of the potential that lies in the offshore wind farms. This ability to generate so high volumes of power is very attractive and so, recently we are noticing a huge increase in the offshore investments with a lot of new technology research that naturally follows the trend.

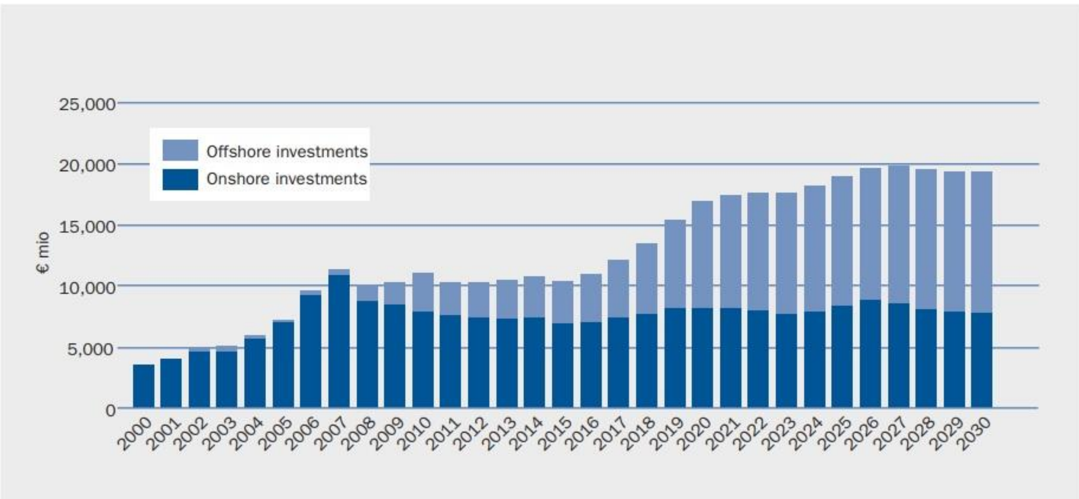

Figure 5 Wind Energy Investments (with projections) in euro per month [13]

The summary of pros and cons of each wind turbine type is presented below [14]:

<u>Onshore wind turbines</u>

pros:

- significantly lower infrastructure costs
- the ease at which the generated electricity can be connected to the grid
- low maintenance costs

cons:

- troublesome noise emission
- disruption of the natural landscape
- higher terrain costs
- onshore wind farms offer less predictability in power output due to the wind speed higher instability

<u>Offshore wind turbines</u>

pros:

- winds on the sea are faster and more predictable therefore the offshore wind farms are offering higher and more reliable power output
- there is no risk of discontent due to altering the visual landscape and the noise pollution
- since noise pollution can be in this case disregarded the wind turbines are allowed to be bigger and therefore they can generate more electricity
- the terrain costs are lower

cons:

- significantly higher costs of installation and maintenance of the infrastructure
- wind turbines have to work under more extreme conditions (corrosive salt water, higher wind speeds)

For now offshore technology, although delivering more power is still not as profitable as onshore due to high construction and maintenance costs. The situation is however rapidly changing and each year we see more and more cost-effective offshore wind farms. While offshore wind power is expensive to extract now, it also offers huge potential for the future world wind market.

The DTU 10MW wind turbine that was chosen as the test case for the purpose of this engineering thesis is classified and designed as an offshore wind turbine, however it can also be used as an onshore wind turbine and still work correctly. The versatility of this turbine model was one of the deciding factors in the test case choice. The tested wind speed range also includes this versatility presenting the wind speeds that are corresponding to both onshore and offshore weather conditions. It is also important to mention that the algorithm developed for the purpose of this study works well both for the onshore and offshore wind turbines.

2. Theory and derivations

This chapter contains descriptions and derivations for equations for the Momentum Theory (MT) and the Blade Element Theory (BET), together with the combination of these two theories called the Blade Element Momentum Theory (BEM). The derived formulas will be then used in other chapters as a theoretical basis for the practical solution application.

2.1 Momentum Theory

In the most simplistic definition the Momentum Theory (MT) deals with the concept of momentum of the fluid flowing through the wind turbine rotor (approximated into a simplified form of a disk). MT is based on the application of conservation laws of fluid mechanics according to which the quantity called momentum that characterizes motion of the fluid never changes in an isolated system. Since MT is derived not only from the Bernoulli equation but also from the Navier-Stokes equations it can be a very broad topic when expanded into details. In the interest of remaining in the scope of this engineering thesis, the following sections will describe only the elements of the Momentum Theory which are the basis of the Blade Momentum Element Method.

Assumptions

Momentum Theory relies on the concept of the actuator disk of axisymmetric loading which can be regarded as a lifting-line model of a rotor with an infinite number of blades. In other words, assuming that the number of blades B will keep increasing to infinity, the area dA taken out by the blades material will keep increasing until it will be equal to the area A of a disk swept out by the path of the blades of a rotating rotor. Since the blades and the hub are simplified into this disk, they are assumed to be infinitely stiff and will not experience any vibrations or mechanical stresses as they normally would in practice.
Equations in this theory are derived under the assumption that the flow is steady and the fluid (in this case the air) is incompressible, homogeneous and inviscid, therefore the whole system is assumed to work without any friction.
Another assumption is that far ahead and far behind the actuator disk the static pressure is identical to the static pressure of the free airstream and that there is a distinct streamline separating the flow passing through the actuator disk under investigation from the surrounding airflow [15, p. 157].
Taking those assumptions into consideration the airstream going through the wind turbine can be simplified into following form:

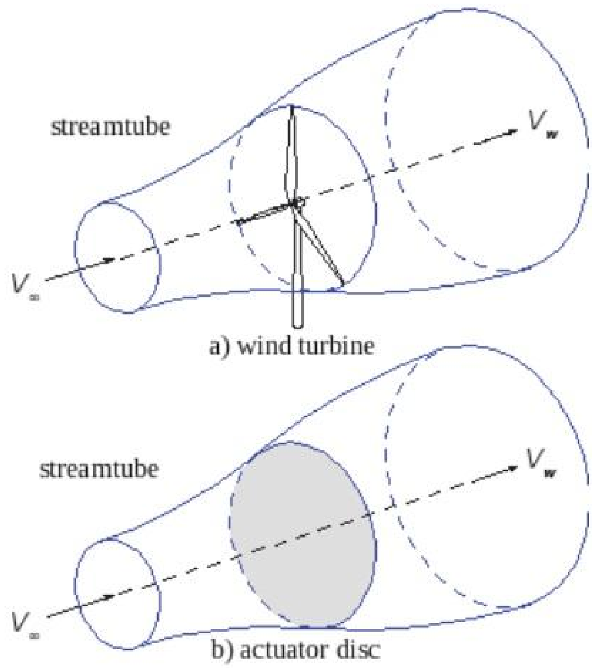

Figure 6 Flowfield of a wind turbine approximated into a form of an actuator disc [16]

Axial Force [17, p. 6]

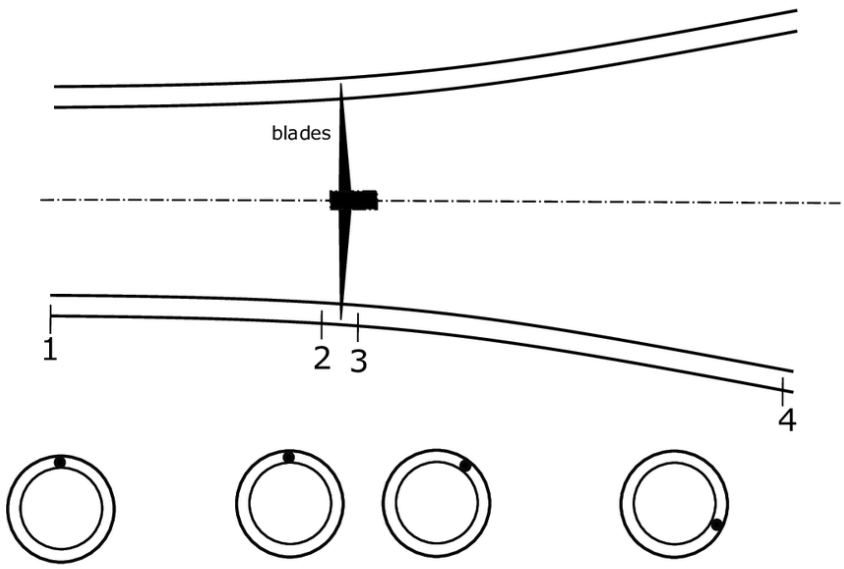

Figure 7 Rotating annular stream tube at various streamwise positions [18]

Figure 7 presents the concept of an annular stream tube, with 4 distinct points marked on the diagram. Point 1 with its corresponding stream velocity V_1 and pressure p_1 is located far upstream of the turbine, point 2 (with corresponding velocity and pressure V_2 and p_2) located just before the actuator disc (blade), point 3 located right after the disc, and point 4 located far downstream of the turbine. Since we assumed $p_1 = p_2$ and $V_2 = V_3$ and also the lack of friction in the system we can use Bernoulli 's equation:

$$p_2 - p_3 = \frac{1}{2}\rho(V_1^2 - V_4^2) \tag{2.1.1}$$

Since $\Delta p = \frac{dF}{dA}$ one can write that:

$$dF_n = (p_2 - p_3)dA \tag{2.1.2}$$

$$dF_n = \frac{1}{2}\rho(V_1^2 - V_4^2)dA \tag{2.1.3}$$

After introduction of the axial induction factor a one can derive the Momentum Theory formula for the axial force:

$$a = \frac{V_1 - V_2}{V_1} \tag{2.1.4}$$

$$dF_n = \frac{1}{2}\rho V_1^2[4a(1 - a)]2\pi r dr \tag{2.1.5}$$

Although correct, the formulas (2.1.4) and (2.1.5) are not yet sufficient for the purpose of being used in the BEM algorithm. Some algebra will have to be made including formulas from the Blade Element Theory to derive them in such a form that will allow the algorithm to work in a way it was intended to. This will be further investigated in the following sections.

Rotating Annular Stream tube [17, p. 7]

The principle of conservation of the angular momentum states that there exists no such system that would not have constant angular momentum. One can consider this principle for the case of the annular stream tube presented in Figure 7. A moment of inertia of a ring is:

$$I = mr^2 \tag{2.1.6}$$

And the the formula for the angular momentum L is:

$$L = I\omega \tag{2.1.7}$$

Where ω is the wake rotation angular velocity. The torque T can be then expressed as:

$$T = \frac{dL}{dt} = \frac{dI\omega}{dt} = \frac{d(mr^2\omega)}{dt} = \frac{dm}{dt}r^2\omega \qquad (2.1.8)$$

When only a small annular element will be considered, it will have corresponding torque equal to:

$$dT = d\dot{m}\omega r^2 \qquad (2.1.9)$$

where, $$d\dot{m} = \rho A V_2 = 2\pi\rho r dr V_2 \qquad (2.1.10)$$

It can then be derived that:

$$dT = 2\pi\rho r dr V_2 \omega r^2 = 2\pi\rho V_2 \omega r^3 dr \qquad (2.1.11)$$

And after introducing angular induction factor a’ one can obtain following derivation:

$$a' = \frac{\omega}{2\Omega} \qquad (2.1.12)$$

$$dT = 4a'(1 - a)\rho V \Omega r^3 \pi dr \qquad (2.1.13)$$

This equation can be then transformed after some algebra into a form more fitting for the BEM equations derivation that will be conducted later [15, p. 186]:

$$dT = \frac{1}{2}\rho V_1^2 (2\pi r dr) r [4a'(1 - a)\lambda_r] \qquad (2.1.14)$$

$$dT_h = \frac{1}{2}\rho V_1^2 (2\pi r dr)[4a(1 - a)] \qquad (2.1.15)$$

Where the local tip speed ratio (TSR) used in the BEM algorithm can be defined as:

$$\lambda_r = \frac{\Omega r}{V} \qquad (2.1.16)$$

The tip-speed ratio is related to the efficiency of the wind turbine. It is worth mentioning that it is also a good indicator of how strong the blade should be, because working with high levels of TSR results in increased centrifugal forces.

2.2 Blade Element Theory

The Blade Element Theory (BET) is a very useful tool that allows to calculate the loads exerted on the blade based on the 3 main factors:

1) **the geometry of the blade**
2) **the airfoils with their Cd, Cl and Cm coefficients**
3) **wind flow velocity vector**

In BET the blade is discretized along its length into N number of independent elements and the length of each obtained section does not change. As it is often the case the accuracy of this approach depends greatly on the number of made divisions- the more elements are taken into consideration, the more accurate results will be produced.

Assumptions

BET relies on the assumptions that there are no aerodynamic interactions between different blade elements (also called sections or radial positions) and that the forces on the blade are solely determined by the lift, drag and moment coefficients. In addition to that the inflow at the blade is assumed to be known and a lifting-lane assumption is used [15, p. 143].
We have to use the lifting-lane assumption, because due to the influence of the airfoil and its wake the flow varies greatly about a blade cross-section. In result it is not generally possible to define a reference local velocity and the angle of attack at a given cross-section [15, p. 118].
The lifting-lane is assumed to lay at the aerodynamic center of the blade, approximately at the quarter of the chord and it is also assumed that the reference local velocity can be evaluated at this line [15, p. 100].

The local solidity

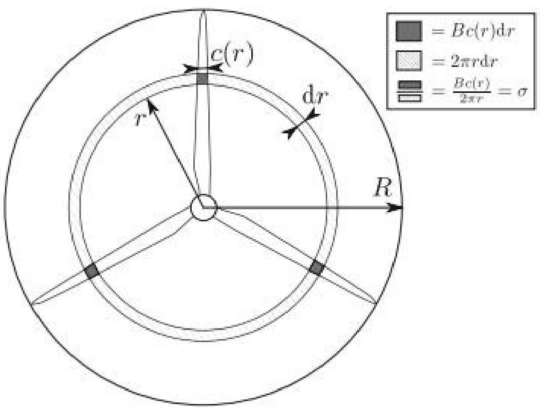

Figure 8 Visualisation of the local solidity. For this particular illustration to be valid, the twist of the blade is assumed to be zero [15, p. 119]

Solidity is an important parameter in the study of rotors which describes how much area is occupied by the blades with respect to the area of the annular stream tube. The solidity changes with the radial positions of the blade and therefore is local. The local solidity is different from the total solidity of the rotor. In the theoretical situation with the infinite number of blades ($B \longrightarrow \infty$) the rotor would have the solidity equal to 1.
The local solidity can be mathematically described as a function of radial position:

$$\sigma(r) = \frac{Surface\ blade\ elements\ area}{Surface\ annular\ element\ area} = \frac{BdS(r)}{2\pi r dr} = \frac{Bc(r)}{2\pi r} \qquad (2.2.1)$$

The Blade Element Model

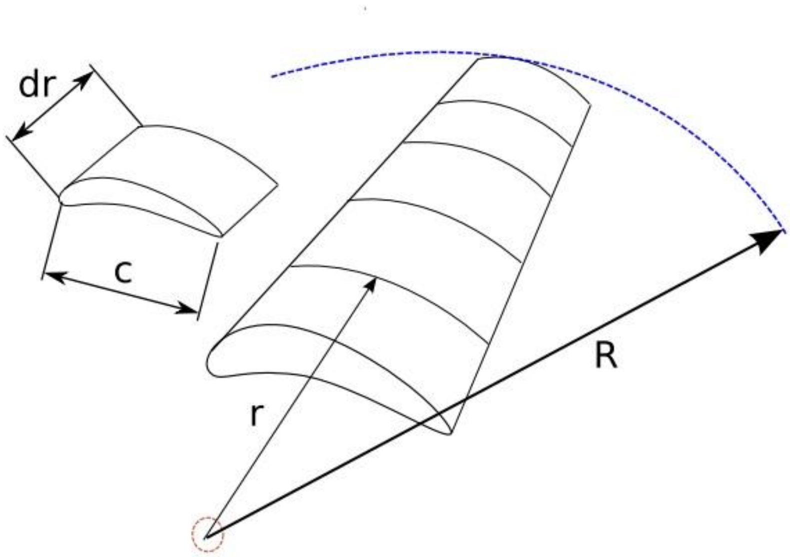

Figure 9 The Blade Element Model visualised [17, p. 8]

The discretized blade consists of N elements each of them located at the radius r from the centre of rotation. Since the radius r is different for each radial position they will all experience different rotational speed Ωr. This combined with the fact that each section can have different chord length c and different twist angle θ means that the flow will be different around each section and therefore they will experience different loads.
As it was mentioned before, the fundamental concept of the BET is to divide the blade into a sufficient number of sections. Then it is possible to calculate the forces at each section and finally to perform a numerical integration of the performance characteristics along the span of the blade. Although most blades are usually divided into 10 - 20 elements the test case investigated in this engineering thesis deals with the blade divided into 39 radial positions, which provides much better accuracy at the cost of the increase in the computational time.

Airfoil

The aerodynamic force exerted on each section of the blade is the result of the lift phenomenon. In the vicinity of the airfoil the air is curving in such a way that the pressure on one side is lower than on the other side and therefore the lift force is generated.

The characteristics of the airfoil can change along the blade length. Usually an airfoil can be approximated as cylindrical in sections that are close to the hub, but it takes a more complex shape closer to the tip. Those inner sections usually do not contribute a lot to generation of power and the cylindrical shape is cheaper to produce and stronger mechanically.
In order to calculate the forces exerted on each section it is important to know the lift and drag coefficients Cl and Cd. When the angle of attack is known one can read their values from the special database or from a graph. The wind turbine that is the focus of this engineering thesis uses cylindrical shaped airfoil near the hub and the FFA-W3 airfoils with various thickness in parts of the blades that are away from the hub. FFA-W3 airfoils are designed and manufactured at The Aeronautical Research Institute of Sweden [19, p. 76 and 85]. Below presented is the FFA-W3-301 airfoil which was used for the middle sections of investigated blades. The number 301 in its name indicates that this airfoil has a thickness of 30.1% of its chord length.

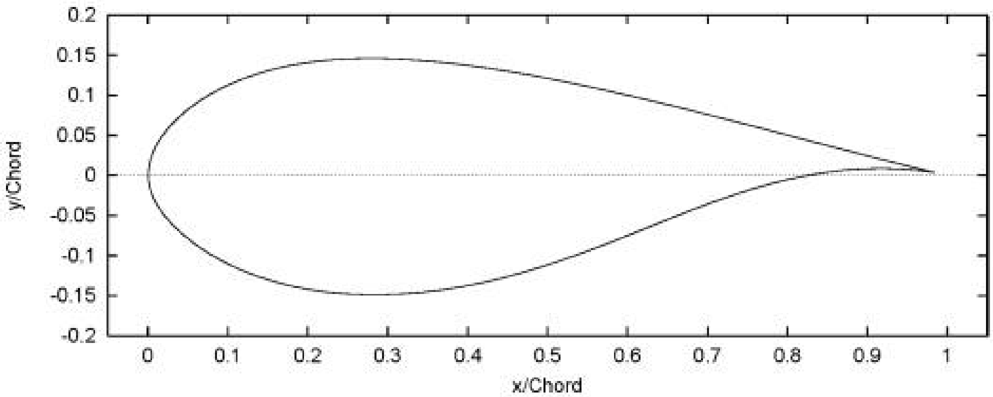

Figure 10 FFA-W3-301 airfoil [19, p. 90]

Wind velocity diagram

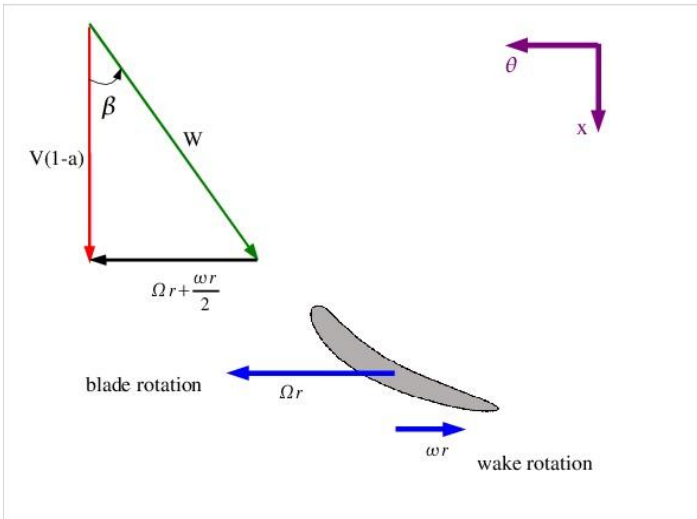

Figure 11 Flow onto the turbine blade represented in form of a wind velocity diagram [17, p. 9]

Since the data of Cl and Cd for different angles of attack is provided for a stationary airfoil tested in the wind tunnel one has to remember that the values provided in the tables or in the charts are obtained for the airfoil that is fixed in a given frame of reference. In reality, however, the airfoil is moving due to the blade rotation and thus, some translation has to be made in order to relate the airflow over the wind turbine airfoil to that of the stationary test.
This translation is available when based on the velocity diagram presented in Fig. 11. It introduces the concept of the relative flow velocity W into the Blade Element Theory described as:

$$W = \frac{V(1-a)}{cos\beta} \qquad (2.2.2)$$

This relative wind flow can be decomposed into its longitudinal component normal to the rotor U_n and into its transverse component in the rotor plane U_t :

$$U_n = V(1 - a) \qquad (2.2.3)$$

$$U_t = \Omega r + \frac{\omega r}{2} \qquad (2.2.4)$$

The transversal component U_t is describing the rotational velocity of the wind flow. Due to the fact that at the inlet to the blade there is no flow rotation and at the blade outlet the flow rotates with its wake rotational speed ω, the averaged rotational wake flow over the blade is $\frac{\omega}{2}$. This is represented in the Eq. (2.2.4), where U_t is a sum of the blade rotational speed Ωr and the speed of averaged rotational flow of the wake.
It can be moreover mentioned that the wake rotation is considered to be a loss because the rotational energy of the wake does not allow for a greater pressure drop near the airfoil and therefore the lift force L is not increased, nor is the power output of the wind turbine.

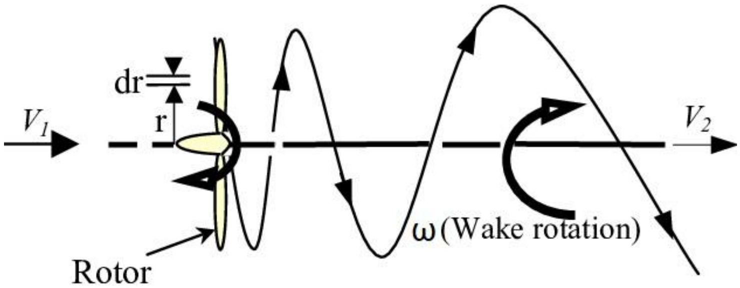

Figure 12 wake rotation visualised [20]

From the wind velocity diagram it can be noted that:

$$\Omega r + \frac{\omega r}{2} = \Omega r(1 + a') \tag{2.2.5}$$

And from the Momentum Theory we know that:

$$V_2 = V_1(1 - a) \tag{2.2.6}$$

Therefore those can be combined into following form:

$$tan\beta = \frac{\Omega r(1+a')}{V(1-a)} \tag{2.2.7}$$

In this equation V represents the velocity of the incoming flow V_1 and β is the relative flow angle onto the blades in the blade element frame of reference. It is important to note that the value of β is not constant between different blade elements and therefore the resulting blade speed will also vary between radial positions of the blade as it is visualised on Figure 13 .

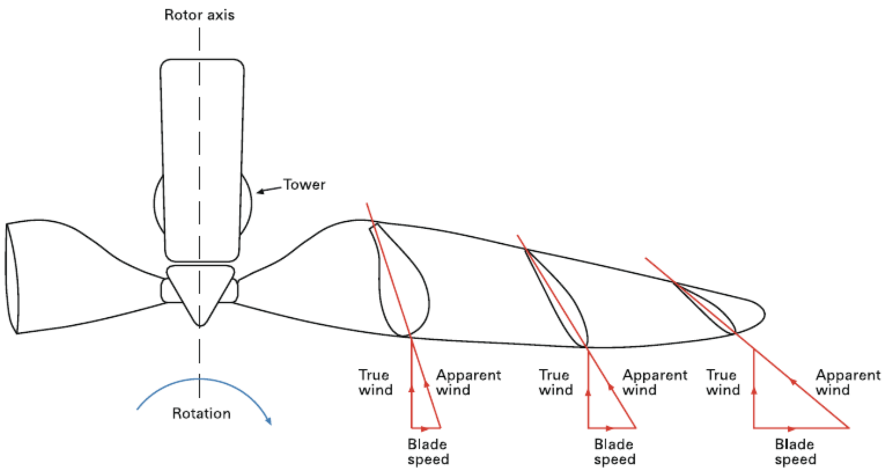

Figure 13 Difference in the blade speed visualised for different radial positions [21]

The relationship between β, the angle of attack α and the airfoil twist angle θ is as follows:

$$\alpha = \beta - \theta \quad (2.2.8)$$

$$\beta = atan\frac{U_n}{U_t} \quad (2.2.9)$$

Based on TSR the expression for tanβ can be written as:

$$tan\beta = \frac{\lambda_r(1+a')}{(1-a)} \quad (2.2.10)$$

Forces on the turbine blade

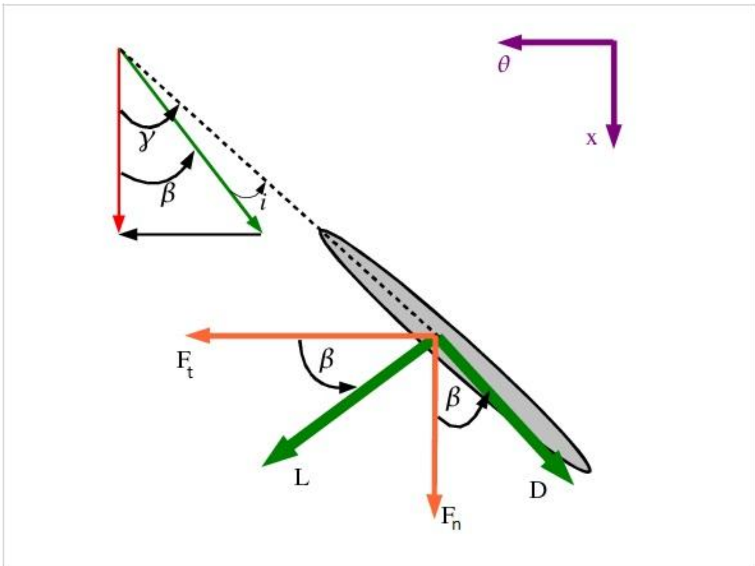

Figure 14 Force components on the blade [17, p. 10]

From Figure 14 it can be observed that the drag force D is always parallel to the incoming airflow, while the lift force L is always perpendicular to it. Axial and tangential force components that are acting on the blade element (radial position) can be then represented as:

$$dF_t = dLcos\beta - dDsin\beta \quad (2.2.11)$$

$$dF_n = dLsin\beta + dDcos\beta \quad (2.2.12)$$

Lift force dL acting on the blade element and drag force dL acting on the blade element can be determined from the definition of the lift and drag coefficients Cl and Cd:

$$dL = Cl\,\frac{1}{2}\rho W^2 c\,dr \qquad (2.2.13)$$

$$dD = Cd\,\frac{1}{2}\rho W^2 c\,dr \qquad (2.2.14)$$

It can therefore be derived that for the B blades the following relation occurs:

$$dF_n = B\frac{1}{2}\rho W^2 (Cl\,sin\beta + Cd\,cos\beta)\,cdr \qquad (2.2.15)$$

$$dF_t = B\frac{1}{2}\rho W^2 (Cl\,cos\beta - Cd\,sin\beta)\,cdr \qquad (2.2.16)$$

It will be also beneficial for future BEM derivations to express dF_n and dF_t using normal and tangential aerodynamic coefficients:

$$dF_n = B\frac{1}{2}\rho W^2 c\,c_n\,dr \qquad (2.2.17)$$

$$dF_t = B\frac{1}{2}\rho W^2 c\,c_t\,dr \qquad (2.2.18)$$

The torque dT can be expressed as:

$$dT = r\,dF_t = B\frac{1}{2}\rho W^2 (Cl\,cos\beta - Cd\,sin\beta)crdr \qquad (2.2.19)$$

$$dT = r\,dF_t = B\frac{1}{2}\rho W^2 cr\,c_t\,dr \qquad (2.2.20)$$

Similarly, the thrust can be expressed as:

$$dT_h = B\frac{1}{2}\rho W^2 c\,c_n\,dr \qquad (2.2.21)$$

The effect of a drag force on a performance of the wind turbine is well visible in those equations because of the increase in thrust and decrease in torque and subsequently power output. Since the BEM method relies heavily on induction factors one can also find it useful in some applications to express dF_n and dT as:

$$dF_n = \sigma\pi \frac{V^2(1-a)^2}{cos^2\beta} (Cl\, sin\beta + Cd\, cos\beta) r dr \qquad (2.2.22)$$

$$dT = \sigma\pi \frac{V^2(1-a)^2}{cos^2\beta} (Cl\, cos\beta - Cd\, sin\beta) r^2 dr \qquad (2.2.23)$$

2.3 Blade Element Momentum (BEM) Theory

In 1926 H.Glauert introduced the Blade Element Momentum Method, which provided a framework for modeling the aerodynamic interactions between the rotor of a wind turbine and the air flowing through it [22, p. 1]. Since this method requires a lot of computational power to provide precise and reliable results it was very hard to utilize it correctly when it was first introduced. The BEM method however gained more and more popularity with the development of computers and now it is widely used for the purpose of estimating the wind turbine efficiency or as a design aid. The blade element momentum method is a combination of the Momentum Theory and the Blade Element Theory. One of the most important practical applications of this method is that it can be utilised to develop a BEM code- an algorithm that can be used to determine the performance parameters of a given wind turbine rotor under some known environmental conditions.
From the Blade Element theory BEM inherits the concept of independent radial positions located across the span of the blade for which the local forces are being calculated. On the other hand from the Momentum theory BEM inherits the influence of the angular momentum on the system. This influence is essential for the working of the BEM model, because it assumes that the energy is extracted when the rotor blades are rotating due to the interaction with the flowing air (transition of momentum from the wind onto the blades).

Assumptions

Although very useful, BEM theory also has some limitations and assumptions that have to be made for the investigated system. As mentioned earlier, the BEM theory is based on two other theories (Momentum theory and Blade Element theory) and therefore BEM inherits those assumptions and limitations from them.

The primary assumption is that the system is static- it means that the airflow field around the airfoil is assumed to always be in equilibrium and the passing flow acceleration is instantaneous.

Another significant limitation is that BEM theory fails when the blade experiences significant deflection out of the rotor plane. As it was mentioned in the section describing momentum theory, it is assumed that the momentum is perfectly balanced in a plane parallel to the rotor plane and any deflections of the rotor will result in the lack of proper accuracy in the aerodynamic modeling of the system [23, p. 2].

Since in Blade Element Theory it is assumed that the forces acting on the element of the blade are two-dimensional, the spanwise flow in models based on the BEM theory is disregarded. It means that there is almost no spanwise pressure variation creating the spanwise flow, and therefore BEM theory based models are losing accuracy when dealing with heavily loaded rotors with big pressure gradients across the span.

BEM equations

It is important to start derivations by mentioning that there are a number of different methods to express the relative wind velocity W and therefore there exists a number of different formulations of BEM equations.
The derivation of BEM equations is based on the comparison of thrust and torque formulas from the Momentum theory and the Blade Element theory.
From Eq. (2.2.20) and Eq. (2.2.21) we know that the torque and thrust in BET are expressed as:

$$dT_{BET} = B\frac{1}{2}\rho W^2 cr\, c_t\, dr \qquad (2.3.1)$$

$$dT_{h\,BET} = B\frac{1}{2}\rho W^2 c\, c_n\, dr \qquad (2.3.2)$$

And from Eq. (2.1.14) and Eq. (2.1.15) we know that the torque and thrust in MT are:

$$dT_{MT} = \frac{1}{2}\rho V_1^2 (2\pi r dr) r [4a' (1 - a)\lambda_r] \qquad (2.3.3)$$

$$dT_{h\,MT} = \frac{1}{2}\rho V_1^2 (2\pi r dr)[4a (1 - a)] \qquad (2.3.4)$$

Since $dT_{BT} = dT_{MT}$ and $dT_{h\,BT} = dT_{h\,MT}$ one can derive that:

$$\frac{W^2}{V_1^2}\sigma c_n = 4a(1 - a) \qquad (2.3.5)$$

$$\frac{W^2}{V_1^2}\sigma c_t = 4a'(1 - a)\lambda_r \qquad (2.3.6)$$

The influence of the tip-loss factor can be shown by modifying those equations into following form:

$$\frac{W^2}{V_1^2}\sigma c_n = 4a\,F(1 - a) \tag{2.3.7}$$

$$\frac{W^2}{V_1^2}\sigma c_t = 4a'F(1 - a)\lambda_r \tag{2.3.8}$$

Those equations can be simplified using following expressions:

$$W^2 sin^2\beta = V_o^2(1 - a)^2 \tag{2.3.9}$$

$$W^2 sin\beta cos\beta = V_0\,\Omega\,r\,(1 - a)\,(1 + a') \tag{2.3.10}$$

And after some amount of algebra the following expressions can be derived:

$$a = \frac{1}{\frac{4\,F\,sin^2\beta}{\sigma\,c_n}+1} \tag{2.3.11}$$

$$a' = \frac{1}{\frac{4\,F sin\beta\,cos\beta}{\sigma\,c_t}-1} \tag{2.3.12}$$

3. Problem definition

Task description

The main task of this engineering thesis is to find the best wind turbine behavior for the test case in terms of pitch angle adjustment and rotor speed adjustment for a set of environmental conditions, so that the resulting power output will be maximized in comparison with the data collected by the DTU company and the Creadis company [24].

Firstly though, the BEM code will have to be tested to check if it is able to produce reliable results to determine whether the algorithm allows for precise predictions that will not be significantly different from the real-life turbine energy output.

The test will therefore be conducted by running the program with the same exact input parameters as are suggested by the DTU, and then the resulting power and thrust values will be compared with the referential data coming from Cradis company and DTU company.

After making sure that the results produced by this BEM algorithm are reliable, the second stage of this study will commence, which is the calculation of optimal pitch and rotor speed for different wind values. Finally the results will be plotted in the form of pitch = f(λ) and then the increase in the annual energy production (AEP) will be estimated using the probability distribution plot called the Weibull Curve.

Test case

While this procedure can be followed for a variety of different wind turbines, a test case of DTU 10MW Wind Turbine placed offshore close to Hel was chosen to be investigated. The most important parameters of this turbine are listed below:

- **Rotor orientation:** upwind with clockwise rotation (it means that it has the rotor facing the wind. This is the case for the vast majority of wind turbines)
- **Cut in wind speed:** 4 m/s (the wind speed at which the blades start rotating)
- **Cut out wind speed:** 25 m/s (the wind speed at which the turbine has to shut down to avoid damage)
- **Rated wind speed:** 11.4 m/s (the wind speed limit at which maximum power is generated - exceeding the rated wind speed will not generate additional power)
- **Rated power:** 10 MW
- **Number of blades:** 3
- **Rotor diameter:** 178.3 m
- **Hub diameter:** 5.6 m
- **Hub height:** 119.0 m
- **Drivetrain:** Medium speed, Multiple-Stage Gearbox
- **Gearbox ratio:** 50
- **Minimum Rotor Speed:** 6.0 rpm
- **Maximum Rotor Speed:** 9.6 rpm
- **Rotor mass:** 227.962 kg
- **Nacelle mass:** 446.036 kg
- **Tower mass:** 628.442 kg

Each blade is divided into 39 radial positions described by the following parameters:

airfoil	shape
I	cylinder
II	FFA_W3_241
III	FFA_W3_301
IV	FFA_W3_360
V	FFA_W3_480
VI	FFA_W3_600

Table 1 airfoil shapes used in the test case

element	r [m]	c [m]	twist [deg]	airfoil
1	4	5.38	14.49106	I
2	4.8	5.38	14.49106	I
3	7.51	5.42	14.42421	I
4	9.1	5.50	14.26041	I
5	10.3	5.58	14.04389	II
6	11.7	5.68	13.58134	II
7	13.3	5.80	12.90876	II
8	15.1	5.94	11.90899	II
9	17.1	6.06	10.68037	III
10	19.3	6.16	9.479599	III
11	21.6	6.20	8.385097	IV
12	24.2	6.19	7.606940	IV
13	26.9	6.13	6.957972	IV
14	29.6	6.01	6.369216	V
15	32.6	5.85	5.809149	V
16	35.5	5.66	5.254826	V
17	38.6	5.43	4.691180	V
18	41.7	5.19	4.087893	VI
19	44.9	4.93	3.445390	VI
20	48.1	4.66	2.790276	VI
21	51.3	4.38	2.125135	VI
22	54.4	4.11	1.481714	VI
23	57.6	3.84	0.868340	VI
24	60.6	3.59	0.294717	VI
25	63.6	3.34	-0.22119	VI
26	66.5	3.12	-0.70484	VI
27	69.3	2.9	-1.12052	VI
28	72	2.7	-1.51714	VI
29	74.5	2.53	-1.87601	VI
30	76.9	2.36	-2.21046	VI
31	79	2.2	-2.52039	VI
32	81	2.02	-2.79797	VI
33	82.9	1.81	-3.03281	VI
34	84.4	1.54	-3.22097	VI
35	85.8	1.14	-3.36962	VI
36	87	1.14	-3.42796	VI
37	87.9	1.14	-3.42796	VI
38	88.6	1.14	-3.42796	VI
39	89.15	1.14	-3.42796	VI

Table 2 Radial positions of the test case blade along with their design parameters

Environmental conditions

Although there are some exceptions to this rule, most offshore wind turbines are located where the water depth does not exceed 20 km. This is visualised by the following heatmap:

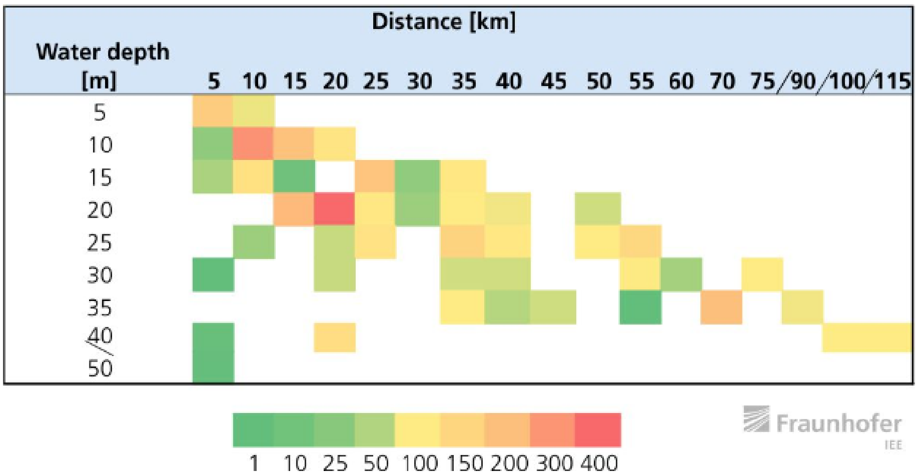

Figure 15 Heatmap of installed offshore wind turbine locations by water depth and distance from the coast [25]

Since the sea around Hel is getting deep relatively close to the shore, the wind turbine that is the test case of this thesis should be placed approximately 5 km away from the coastline.

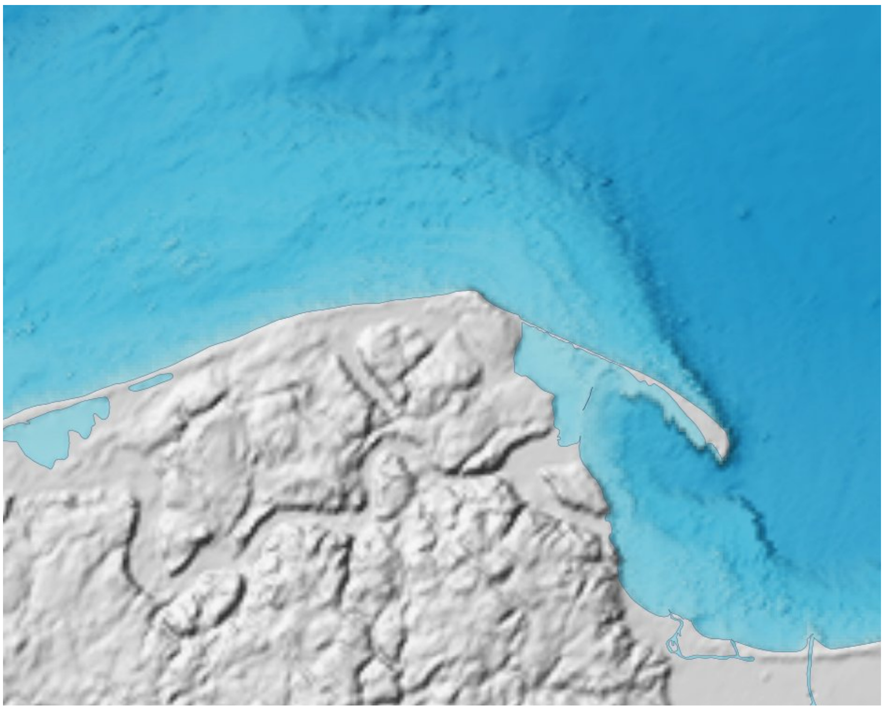

Figure 16 The depth of water near the Polish coastline. The light blue color indicates more shallow areas, while the dark blue indicates deep sea [26]

In order to estimate the annual energy production of the wind turbine placed in this region the area surrounding Hel has been investigated in terms of average wind speed at the height of 100m. It resulted in conclusion that the wind speed with the biggest probability of occurring is $7\frac{m}{s}$.

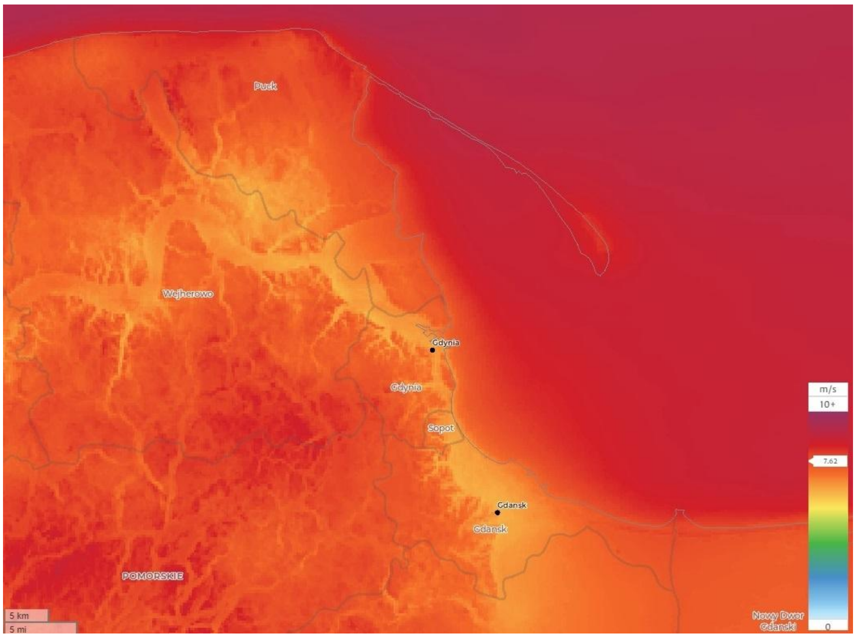

Figure 17 wind velocity distribution visualised for Polish coastline [27]

Expectations and result possibilities

While the improvement of efficiency by 0.1% might seem negligible, in reality it would contribute to huge amounts of extra power generated annually and therefore it would greatly increase the profitability of the investment for the company. Therefore I do not expect a huge difference in efficiency for the optimized behavior in comparison to the normal one, but even a very small improvement will be deemed as satisfactory. Given the fact that the available wind turbines are shaped in the course of long studies and this current analysis relies on a number of assumptions and approximations, it should be expected that any potential improvements made to the power output will not be perfect.

In the case of absence of any improvement, the secondary task of this thesis will be to investigate why this situation occurred and what can be done to change it. Perhaps it can be the case that already existing behavior instructions for this turbine are already optimized beyond the possibilities of the BEM algorithm utilized in this work and in such a situation the conclusion can be drawn that the improvement in the efficiency has to be made in other aspects of operation of the wind turbine. It would mean that there will be no more significant improvement of efficiency by adjusting the pitch and rotor speed, and the focus for further investigations should be placed on better choice of materials or better geometry of the blades.

4. Solution

BEM algorithm

Before writing an actual matlab BEM code it is important to firstly highlight logical dependencies and the overall order of the procedure. This is done in a form of the BEM algorithm which describes an iterative process of determining performance properties of each discretized radial position and then integrates them over the length of the blade. For some applications non-linear solution techniques can be utilized instead of an iterative process, however in most cases it is more practical to use a simple iterative process where each iteration is looped until proper convergence is achieved. The quality of calculated results can be additionally enhanced by introduction of the tip-loss factor F into the algorithm. The effect it has will be visualised in Figure 18.

In essence, the BEM algorithm looks like this:

Step 1: Definition of the initial input such as the airfoil lift and drag coefficients plots, chord lengths, initial airflow velocity, radius of the blade with division for radial positions, blade twist angle for each radial position and the rotor rotational velocity. In addition to that the algorithm parameters should be defined such as the acceptable tolerance for the induction coefficients and the maximum acceptable amount of iterations, because it is usually not a good idea to let the programme loop indefinitely without a safety mechanism.

Step 2: Initial guess of the induction coefficients. While it is possible to start from a value of 0 it is recommended to use one of many possible approximation formulas, since it will decrease the computation time. The trial and error testing of various approaches has shown that the approximation a = 0.3 and a '= 0 is both sufficient and simple enough for this purpose.

Step 3: The loop begins in this step. It consists of computation of the normal and tangential components of flow velocity using formulas derived in previous sections:

$$U_n = V_0\ (1 - a) \tag{4.1.1}$$

$$U_t = \Omega r\ (1 + a') \tag{4.1.2}$$

Step 4: Determination of the flow angle. Determination of the tip-loss factor (if variable).

$$\beta = atan\frac{U_n}{U_t} \tag{4.1.3}$$

$$F = \frac{2}{\pi} acos e^{-(\frac{B}{2}\frac{R-r}{r\, sin\beta})} \tag{4.1.4}$$

Step 5: Determination of the angle of incidence

$$\alpha = \beta - (\theta + pitch) \tag{4.1.5}$$

Step 6: Take the value of lift coefficient and drag coefficient for obtained angle of incidence from the database or read it from the plot. Interpolation of value is recommended since even small changes in the argument (the angle of attack) will result in significant change in Cl and Cd coefficients.

Step 7: Use Cl and Cd to calculate the normal and tangential aerodynamic coefficients c_n and c_d

Step 8: Save the previous values of induction coefficients, and compute new ones using obtained coefficients c_n and c_t for comparison

Step 9: Compare new and previous induction coefficients and go back to step 3 if the induction coefficients have not converged within the allowable tolerance. Otherwise it is the end of the loop for this radial position.

Step 10: Compute power for this radial position and save the value

Step 11: Repeat the procedure starting from step 2 for each radial position along the blade

Step 12: Integrate calculated power values along the blades. The result will be the total power output of the wind turbine.

The optimization of generated power makes sense only for wind values that are below rated velocity. Once the wind speed reaches this threshold, the power output does not need any improvement - on the contrary, it needs to be reduced as otherwise the wind turbine may be damaged. Therefore, the algorithm will be used only for wind values in the range $4\frac{m}{s}$ - $11\frac{m}{s}$.

Another aspect that should be considered is which ranges of pitch angle and rotor velocity will be taken into consideration, and how big will be the increment between each iteration. While initially a very big range had been taken into consideration it turned out that the vast majority of points were not suitable for wind energy extraction, or were significantly sub-optimal with regards to the baseline parameters. Therefore since the pitch and Ω baseline values had already been established it makes sense to take into consideration only a small region in the vicinity of those points. The tested region is therefore
<pitch - 2° ; pitch + 2°> and **< Ω - 1rpm ; Ω + 1rpm>**. The choice of the increment turned out to be quite problematic, because smaller increments meant better accuracy, but it also meant significantly increased computation time. There were also technical limitations to the actual accuracy of the wind turbine pitch control systems that had to be taken into account. Finally the increment was set to the values of **Δpitch = 0.001** and **ΔΩ = 0.01** .

The BEM code prepared for the purpose of this engineering thesis consists of 3 scripts and is presented below. The red color of the font indicates the input parameter that can be changed to describe different test cases. The values of presented chord, twist angle and radial positions vectors are simplified in comparison to those used in real calculations for which the values were obtained.

[annotation: the code has been classified due to copyright]

5. Results

In order to be assured that the algorithm is working correctly a comparison has been conducted between two versions of the BEM algorithm that was written for the purpose of this engineering thesis, and the data taken from two trusted sources used as a reference. Firstly an algorithm with constant tip-loss factor F = 1 (defined in Eq.4.1.4) was tested but as it can be noticed it provided results that were too high to be accepted. In the second order the algorithm with variable F was tested and it provided much more satisfactory results. The pitch curve obtained during those tests can therefore become a reliable baseline for planned optimizations.

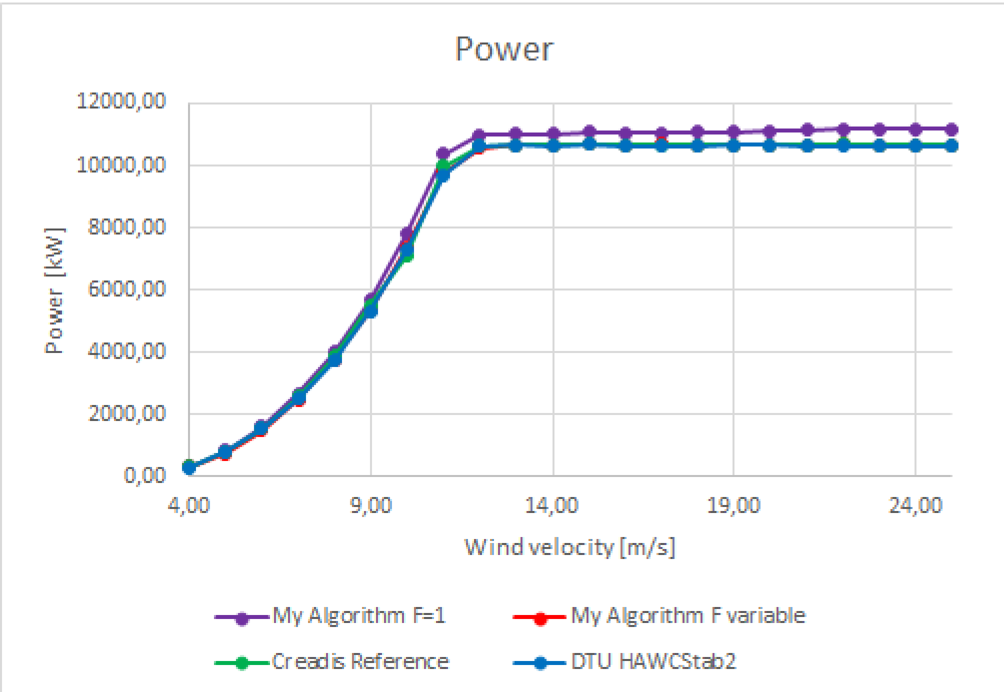

Figure 18 comparison of obtained power values between working BEM code and referential data

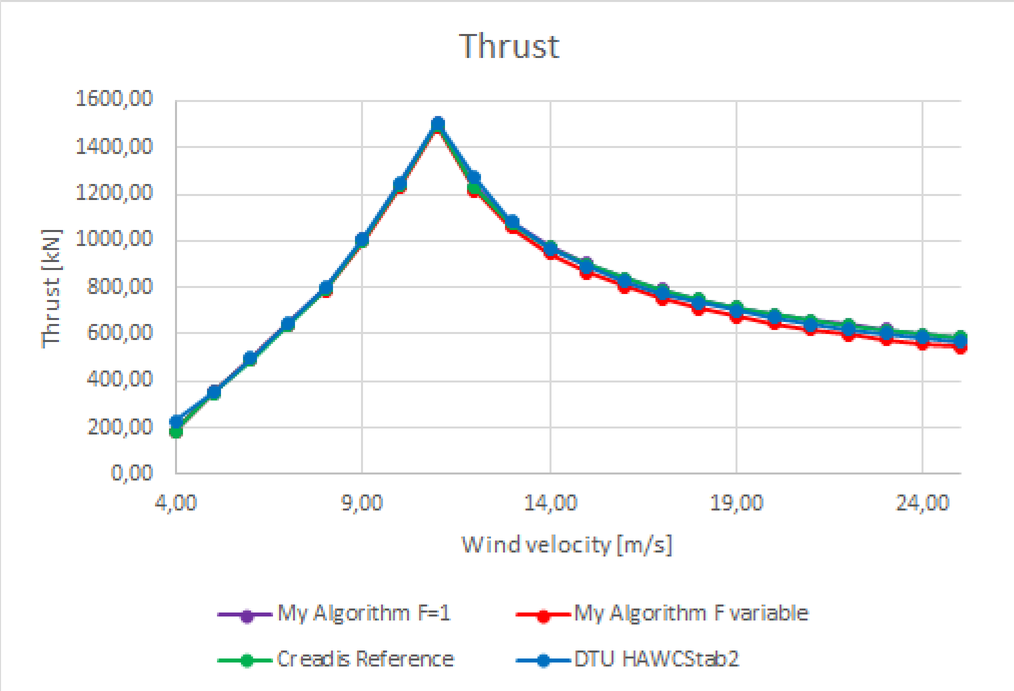

Figure 19 comparison of obtained Thrust values between working BEM code and referential data

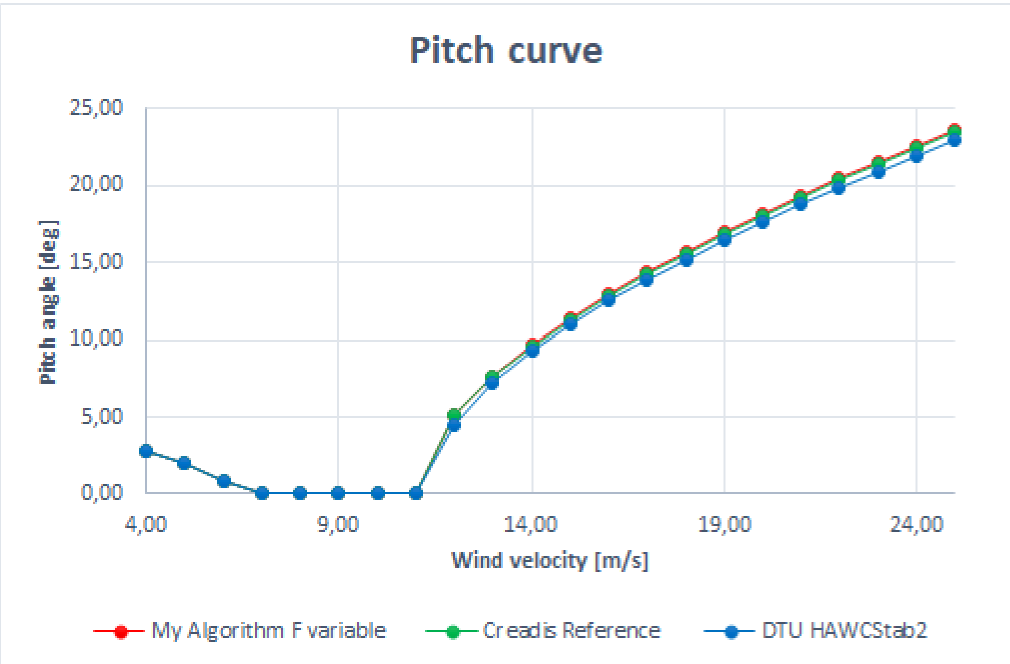

Figure 20 comparison of pitch values between working BEM code and referential data

The next step in the process of optimization of the turbine behaviour would be to estimate the current, non-optimized annual energy production of the test case wind turbine. For this purpose the probability of occurrence of each wind speed described by the Weibull Curve for the area in the proximity of Hel was multiplied by the amount of energy that would be produced annually by a wind turbine working under a given constant wind speed.

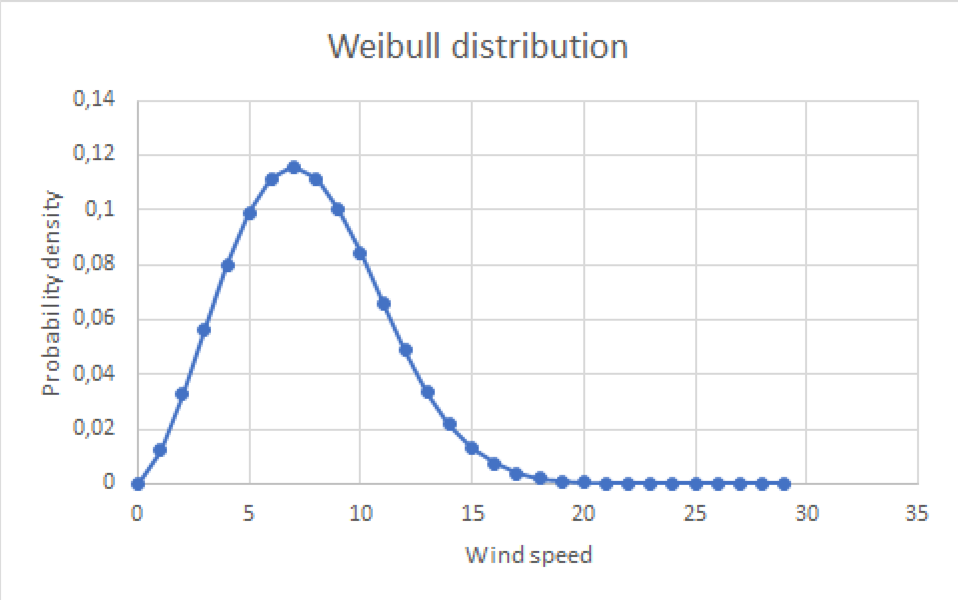

Figure 21 Weibull Curve for Hel

Since in the energy industry it is usually better to give the amount of energy in kWh rather than in joules, the baseline AEP of the wind turbine was estimated to be 36516986 kWh or 36 517 MWh.

Design of the optimized wind turbine controller database

The algorithm provided very interesting results that can be used as an input for the wind turbine controller system managing the turbine behaviour. For lower wind speeds it managed to optimize the power gain significantly (for example extra 60 kW generated for $V_0 = 5\frac{m}{s}$), however for wind speeds that are closer to the rated speed, very little power was generated. Fortunately, this is not very problematic in this case, because the wind turbine that is the test case for this work would not be operating at rated speed very often anyway. The reason for this is that the average wind speed for Hel is $7\frac{m}{s}$ and for most of the time the turbine will operate in the conditions for which the algorithm enhanced significantly the power output.

It is worth mentioning that the baseline rotor speed did not really require much adaptation and all the changes applied to it were small. This coincides well with the observation that the extra power gain is insignificant in cases in which only the rotor speed was being subjected to optimization. On the other hand the baseline pitch angle values were quite far from optimized and therefore the algorithm managed to improve efficiency of the wind turbine significantly for those cases of wind condition.

	Initial wind speed [m/s] V0	pitch [deg]	rotor speed [RPM]	Power output [kW]	Extra Power generated [kW]
baseline	5	1,9700	6,000	759,11	60,22
optimized		3,1100	6,000	819,33	
baseline	6	0,9000	6,000	1491,21	52,21
optimized		2,0590	6,000	1543,42	
baseline	7	0,0000	6,000	2489,75	26,33
optimized		0,6950	6,000	2516,08	
baseline	8	0,0000	6,430	3767,18	0,03
optimized		0,0000	6,420	3767,21	
baseline	9	0,0000	7,229	5363,84	0,02
optimized		0,0000	7,220	5363,86	
baseline	10	0,0000	8,030	7357,82	0,01
optimized		0,0000	8,020	7357,83	
baseline	11	0,0000	8,836	9793,21	0,06
optimized		0,0000	8,830	9793,27	

Table 3 The results of optimization together with baseline values for comparison for sub-rated wind speeds

This database can now serve as a set of instructions for a controller that will influence the behaviour of the turbine in a way that the power output will be optimized.

Knowing the improved power output values one can calculate the total AEP of this wind turbine for Hel, and its AEP gain with regard to the baseline, pre-optimization value. For this purpose the same Weibull Curve will be used as it was for the baseline AEP calculations. Therefore it can be calculated that the annual power gain due to the optimization is **129924.086 kWh** or **129.924 MWh**, which is a significant amount of energy produced annually by a single wind turbine. This means that AEP has improved by **0.3558%**.

6. Discussion

This chapter provides an overall overview of the quality of the project results, the challenges faced during creation and implementation of the solution, and the profitability of the project itself. The content of this section was written on the basis of the discussions that took place during various meetings between the author, Dr. Grzegorz Liśkiewicz, and Dr. Vladimir Leble.

Adjustments to the solution

When performing calculations in an iterative process one has to remember that some operations can be very compute-intensive, as was the case in this project. The quality of the results can be adjusted using different algorithm parameters describing accuracy. There was however a serious issue with the time needed to complete the calculations for the set of possible solutions, because even the relatively small region that was close to the baseline curve was taking too long to solve with high accuracy. The solution was to run the algorithm two times. For the first run the accuracy was unsatisfying, however it provided information about the behavior of the power curve for different sets of arguments. This data was then analyzed, which allowed to extract a much smaller region of interest i.e. the small set of arguments around the maximum value of output power. This set was then inserted into the algorithm and the calculations were performed once again, this time with significantly higher accuracy. This approach is more ecological because it provides much more reliable and useful results without the necessity of running for days or weeks and using a lot of electrical power in the meantime, which fits really well into the theme of this work.

Confirmation of validity

It was very helpful to test the validity of my results by checking whether the Creadis algorithm would also provide optimized power values (with regard to the baseline arguments) for the same input. The test was conducted for all wind velocity values below the rated velocity and it was confirmed that the power was properly optimized. Moreover, the percentage difference between optimized and baseline Creadis power outputs was similar to the one obtained for my algorithm. Finally the baseline and optimized values for both algorithms were analyzed by dr Vladimir Leble who is an expert in the field of wind turbines, and were assessed as acceptable.

The conclusion that can be drawn from this is that the optimization method works correctly and can be trusted to be reliable.

Difficulties met during the process

Due to the many ways in which BEM could be implemented, some obstacles were encountered during completion of this project. It required some additional derivations and tweaking to make different formulas taken from different textbooks and sources correctly work together. Another encountered issue is the arbitrary nature of a number of parameters that could have not been estimated in any other way than experimentally. This is especially true for parameters describing the accuracy and inner-workings of the BEM algorithm. For example, the value of the assumed tolerance for the convergence of the inductance coefficient was selected arbitrarily by trial and error until satisfactory results were obtained. It was then compared to accepted tolerance values used in different BEM algorithms created by different authors to estimate that the reliable results will be produced for the $\Delta a = 10^{-8}$.

The estimation of the tip-loss factor F also proved to be quite troublesome. This number was often calculated by the algorithm to be a complex number and therefore the provided results were unreliable and far from expected values. The fix for this problem turned out to be the safety mechanism that ensures that if the value of $\sin\beta$ will be smaller than 0.01, then F will not be taken into consideration for the given iteration. As it was concluded from analysis of another fix used by Creadis this is most likely connected to the fact that the tip-loss factor should be calculated only for the middle sections of the blade i.e. the first and the last radial position should be calculated with value of F equal to 1.

Annual optimization gains

It can be beneficial to put the obtained annual energy gain into perspective by recalculating it into extra money earned annually. While for different countries there exist different tariffs for energy and different taxation that also influences the gain, in Poland the price is 250 pln per 1kWh [28]. It means that just by improving instructions for the controller each wind turbine will generate an extra 32 481 pln of income each year. The cost of updating the controller instructions is negligible, because all that has to be done is to update the database by sending 1 person to the top of the tower once to upload the data to the computer.

7. Conclusions

Completion of this project allowed for familiarization with the principles of operation of wind turbines and a significant expansion of knowledge in the field of aerodynamics.

Additionally, this work provided a lot of knowledge about the production and installment process of wind turbines both onshore and offshore, along with the advantages and disadvantages of each solution. Finally, it is worth mentioning that participation in this project allowed for greater freedom in the use of programming in Matlab and designing logical structures used in all programming languages.

How satisfactory are the results?

When starting work on this engineering thesis, there was a discussion with an expert about what a given wind energy company would expect from this BEM algorithm, what result would satisfy them and how useful it would be for them. Usually the wind turbines are operating in a group collection called a wind farm and each wind turbine is already producing significant amounts of electrical power. It means that even though the power output improvement of 0.1% would seem insignificant at first, it would contribute to a significant amount of extra income and the power company would be very satisfied with it and very determined to achieve it. The algorithm that was created for the purpose of this work managed to increase this income by 0.3558% and therefore it exceeded expectations triplefold. The result is therefore considered to be satisfactory and the goal of this engineering thesis is achieved.

What are the practical applications of this controller design?

The data collected during the calculations can have many practical applications and can be profitably used as an input database for a 10MW wind turbine controller operating in Hel. Since the environmental conditions are changing rapidly in recent days, there is no guarantee that the wind speed distributions over the considered region will remain the same, and therefore the AEP can be subjected to significant changes. This particular design is more suited to operate in low-wind speed conditions and therefore it could be used to mitigate the power losses in case the wind would decrease its annual average velocity. In the case that the wind speed for this region would increase in the future, then unfortunately this design would prove to be on-pair with the pre-optimized baseline design.

What are practical applications for this developed BEM code?

While the developed BEM code was only used in the course of this work to create an optimized database for controller of the wind turbine test case, it is worth noting that it can be used with great freedom of adaptation to different cases. The code takes a lot of variables as an input, and therefore can be adjusted for different wind turbine models working in different environmental conditions. It will work for wind turbine models equipped with a larger number of blades, having different airfoil design and different blade and rotor dimensions. The quality of the calculations can also be changed in case if better accuracy is required. As this code is universal, it can also be used to verify other algorithms in the same way that the Creadis code was used to validate results obtained during this work.

What are the other advantages provided by the solution of this work?

While certainly welcomed, the extra 32 481 pln of income is not the only advantage that this solution offers. When the society is becoming increasingly dependent on energy it is becoming more and more crucial to improve the efficiency of the clear energy extraction methods to make this energy as cheap and viable as it is possible. The optimization procedure that was developed during this work can be used to significantly increase the viability of the clear energy in the market. The good thing about this is that not only a company that owns the wind turbine will benefit from it, but it could also relate to the lower energy prices for households. It will also make clean energy more competitive in comparison to fossil fuels and therefore the ecological impact of this solution will be positive, because it will result in a decrease of annual CO_2 released to the atmosphere. It is not easy to predict how significant those changes will be in our fight against global warming, however every little effort matters especially in cases like this one, where it costs almost nothing to upload an improved behaviour database into the wind turbine controller.

Recommendations

Right now the presented controller database has been calculated for wind speed values expressed in integers. In reality however the wind speed is never a whole number and it instead takes the form of a fraction. It forces the controller to interpolate between two values obtained for integer arguments and the most common and mathematically viable form of interpolation is linear interpolation. This however can be in some cases highly problematic because the function of power does not increase linearly. For the purpose of this engineering thesis, the increment between the two arguments is satisfactory, however, calculations with additionally reduced increment would give better results, and hence higher power output and annual income, because there would be no power losses due to sub-optimally optimized interpolated values.

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